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This research has developed a number of new results for the modeling and control of spatially continuous dynamic processes. The work includes developing numerical methods for implementing controller designs for those linear and nonlinear partial differential equations that describe the dynamics of fluid/structure systems and thermo-mechanical alloys.

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## COMPUTATIONAL METHODS FOR MODELING AND ACTIVE CONTROL OF DISTRIBUTED PARAMETER SYSTEMS

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by

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## SUMMARY OF RESULTS

The development of mathematical models for use in control design and optimization often requires several stages of approximation. Also, in the area of distributed parameter control some type of numerical approximation must be introduced at some point in the modeling process. We are primarily concerned with the problem of constructing approximation schemes for those linear and non-linear partial differential equations that describe the dynamics of fluid/structure systems and thermo-mechanical alloys. Finite element, Galerkin and finite difference schemes are typically used to "discretize" continuum models, while in the frequency domain one might construct rational approximations of non-rational transfer functions. For computing purposes, state models offer certain advantages in that there are numerous computational algorithms well suited for the matrix-linear algebra problems that occur in control design. Direct discretization of continuum models usually produce state space models as do frequency domain methods when followed by realization schemes. Both approaches have advantages and disadvantages and each approach leads to its own characteristic set of problems. However, regardless of the approach the finite dimensional design model should be robust in the sense that the numerical algorithm used for control design is well-conditioned. It has been known for some time now that numerical algorithms which assume a specific system property such as controllability (e.g. pole placement) or stabilizability (e.g. regulator problems) can expect to become ill-conditioned if the design model is nearly uncontrollable or nearly unstabilizable, respectively.

We have shown that in certain parabolic and hyperbolic problems finite element schemes not only preserve system properties they often provide the most robust design models. We were somewhat surprised by this discovery. In fact, we learned that many finite dimensional design models based on balancing approximations (specifically constructed to be robust) were less robust than straight forward finite element and finite difference models. The classical equations of thermo-elasticity were studied and it was shown that finite element models may produce ill-conditioned control problems. The analysis of numerical conditioning for control of partial differential equations has several facets. One new observation concerns the use of balancing in model reduction techniques. If one uses finite elements to construct a 'raw' finite dimensional model of order  $N$  and then uses balancing to reduce this model to a  $K = N - 1$  dimensional design model, then for large values of  $N$  this process is numerically ill-conditioned. A partial explanation of this observation lies in the fact that most

balancing algorithms require (stable) minimal systems (ie controllable and observable). We have shown that for many control problems involving heat-transfer or elasticity, the approximating systems get close to being non-minimal. On the other hand, some design techniques (eg LQR approaches) are very well conditioned. Hence, LQR design with 'raw' finite-element models is often more practical than some other methods that are based on balancing. ( See [1], [2], [3], [6], [8], [12], [17], [20])

Our work on numerical algorithms for the linear quadratic optimal control problem for systems governed by partial-integro differential equations that model viscoelastic and thermoelastic structures has raised several questions. Convergence of the algorithm has been established for a large class of models. However, the questions of adjoint convergence and preservation of stabilizability uniformly under approximation have not been completely answered. The finite dimensional LQR control problems are numerically well-conditioned. ( See [4], [5], [9], [28], [33], [35]).

We have also completed a study of a control problem for a infinite-dimensional nonlinear system based on Burgers' equation. The method is first to linearize the system and then to employ linear-quadratic regulator theory to design a time-invariant controller. The linearized equation is the heat equation. Numerical approximations were introduced and studied. We proved that under reasonable assumptions the standard finite element scheme preserved stabilizability and detectability uniformly. Moreover, the LQR based control produced a non-linear closed-loop system that enhanced stability and reduced "weak shocks" for a wide range of Reynolds number. In addition to the feedback laws obtained by LQR methods, we considered direct non-linear feedback and applied extended linearization to Burgers' equation with a de-stabilizing term. (See [13], [29], [30], [40], [41]).

One aspect that must be addressed when considering control of highly maneuverable aircraft is the rigid body control problem. Optimal rigid body motions have been investigated in the absence of direct control over one of the angular velocity components. A classification of extremal solutions is presented and time optimal bang-bang solutions are given. These results have application to control of aircraft in low dynamic pressure regime at high angles of attack where the usual aerodynamic control surfaces are ineffective for generating control moments. These results have been extended for a model of the High Angle of Attack Research Vehicle (HARV). Additional problems of optimal flight performance have also been studied. ( See [14], [15], [21], [22], [26]).

Problems of control of external fluid flows are highly non-linear, and even the construction of open loop simulations is extremely complex. At this point we (joint work with R. OU) have a code for 2-D flow about a rotating cylinder and have initiated some optimal control designs. We have already been able to show that there is a solution to the problem of determining the rotation rate that maximizes the lift to drag ratio. Our project with Professor D. Pelletier (Ecole Polytechnique de Montreal) involves the use of Dr. Pelletier's adaptive grid code in conjunction with an optimization code to investigate flow separation control. (See [31], [32]).

The development of computational algorithms and software for use in control design depends on obtaining convergent numerical schemes that preserve important control system properties. Moreover, the numerical conditioning of the approximating control problem is directly related to the measures of stabilizability, controllability and other system properties important in control design. The development of robust approximation schemes that preserve such system properties is fundamental to the solution of the problem of modelling for control design. Although we have made considerable progress in understanding finite element approximations of certain parabolic and hyperbolic systems, considerable work remains to be done on thermo-elastic systems and hybrid systems such as those that model fluid/structure interactions. We also worked on "pre-conditioners" that might improve conditioning by using selected state transformations to increase control system radii. (See [12], [17], [20]).

We have developed state space models and convergent approximation schemes for control of thermoelastic materials and shape memory alloys. These problems are extremely complex in that the basic equations are non-linear. In the case of thermoelastic materials, verification of adjoint convergence for the standard computational schemes has not been established even in the linear case. The basic modeling problem for shape memory alloys remains unsolved. We have studied a thermoviscoelastic model with a singular kernel in the strain memory term. The dynamics were cast in a semigroup setting and a generalized well-posedness result was established for the system with various boundary conditions. Asymptotic stability was also proved. For the case with  $L_2$  kernel the Trotter-Kato theory was used to formulate convergent approximation methods for the system and its adjoint. A time-invariant controller was constructed as the solution to a 'standard' linear-quadratic regulator problem. Numerical computations of the eigenvalues for the system (both open-loop and closed-loop) were carried out to

study the convergence rates. We constructed various models that attempt to capture the complete hysteresis loop that one observes in experiments of loading and unloading these materials. (See [4], [5], [27], [28], [34], [35], [39]).

In some cases the mathematical of a physical process can produce solutions that do not depend on system parameters in a smooth way. This complicates problems of system identification where solutions sensitivities are needed. Thus, we have studied the development of an infinite dimensional quasilinearization algorithm for parameter estimation in models for which the solutions do not depend smoothly on the parameters. The fluid/structure systems and shape memory alloys are providing the motivation for this effort. We have theoretical and computational results for one dimensional quasi-linear systems of the type that are typical of fluid flows. In addition, we established the convergence of a quasilinearization algorithm and applied the algorithm to the problem of estimating unknown delays in functional differential systems. We initiated an effort to extend these ideas to problems non-smooth spatial varying coefficients so that they may be applied to more practical problems. ( See [3] )

We also worked on the construction of design models and computational schemes for various structural and structural/fluid control problems. These problems are characterized by the property that the basic continuum models are coupled systems of mixed partial differential equations, where the coupling occurs through a moving boundary. It has recently been shown (Banks and Wang) that standard finite element schemes do not preserve stabilizability uniformly. This means that completely new computational schemes must be developed if one plans to use these schemes in control design. We made considerable progress on a number of problems of this type. (See [7], [10], [11], [18], [23], [24], [25]).

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1. J.A. Burns and G. Peichl, A Note on the Asymptotic Behavior of Controllability Radii for a Scalar Hereditary System, Proceedings of the International Conference on Control of Distributed Parameter Systems, Graz, Austria, 1988, International Series of Numerical Mathematics, Vol 91, Birkhauser Verlag, Basel 1989, 27-39.

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Z. Liu, "Approximation and Control of a Thermoviscoelastic System", Ph.D. Thesis, Department of Mathematics, Virginia Polytechnic Institute and State University, August 1989. - Assistant Professor, University of Minnesota-Duluth

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K. L. Oates, "A Study of Control System Radii for Approximations of Infinite Dimensional Systems", M.S. Thesis, Department of Mathematics, Virginia Polytechnic Institute and State University, August 1991. - Staff Scientist, Applied Physics Laboratory

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